$e^{-2\alpha d}$ and the phase shift is βd. These simple expressions for the efficiency and phase shift are not valid when the transmission line is not terminated in a matched load.

The load voltage E_L, load current I_L, input voltage E_A, and input current I_A are given by the equations

$$E_L = E_L^+(1 + K_L), \quad (1)$$

$$I_L = (E_L^+/Z_0)(1 - K_L), \quad (2)$$

$$E_A = E_L^+(e^{\gamma d} + K_L e^{-\gamma d}) = E_L^+ e^{\gamma d}(1 + K_A), \quad (3)$$

$$I_A = (E_L^+ e^{\gamma d}/Z_0)(1 - K_A); \quad (4)$$

where E_L^+ is the component of voltage at the load associated with the wave traveling toward the load, the voltage reflection coefficient K_L at the load is

$$K_L = \frac{(Z_L/Z_0) - 1}{(Z_L/Z_0) + 1}, \quad (5)$$

and the voltage reflection coefficient K_A at the input is

$$K_A = K_L e^{-2\gamma d}. \quad (6)$$

It follows from (1)–(4) that

$$\frac{E_L}{E_A} = \frac{1 + K_L}{1 + K_A} e^{-\gamma d} \quad (7)$$

and

$$\frac{I_L}{I_A} = \frac{1 - K_L}{1 - K_A} e^{-\gamma d}. \quad (8)$$

The input impedance Z_A is

$$Z_A = \frac{E_A}{I_A} = Z_0 \frac{1 + K_A}{1 - K_A}. \quad (9)$$

The efficiency η of power transfer is

$$\eta = \frac{E_L I_L \cos \theta_L}{E_A I_A \cos {}^{\alpha}{}_A} = \frac{I_L{}^2 R_L}{I_A{}^2 R_A} = \frac{|1 + K_L|\,|1 - K_L| \cos \theta_L}{|1 + K_A|\,|1 - K_A| \cos \theta_A} e^{-2\alpha d} = \frac{|1 - K_L|^2 R_L}{|1 - K_A|^2 R_A} e^{-2\alpha d}, \quad (10)$$

where θ_L and θ_A are the angles of Z_L, and Z_A, and R_L and R_A are the resistance components of Z_L and Z_A, respectively. The efficiency can be converted into attenuation.

Circular transmission line charts provide a convenient graphical method for evaluating (7)–(10). First the normalized load impedance $Z_L' = Z_L/Z_0$ is plotted on a transmission line chart as shown in Fig. 1. This point is rotated about the center C of the chart through the angle $2\beta d$, which corresponds to d/λ, where λ is the wavelength in the transmission line. The new point is designated Z_A''. The distance $\overline{CZ_A''}$ is measured with any convenient scale and is multiplied by $e^{-2\alpha d}$. The point Z_A' is located on the line CZ_A'' at the distance $\overline{CZ_A''}e^{-2\alpha d}$ from C. (If the transmission line is lossless, $Z_A' = Z_A''$.) Lines are drawn through the points Z_L' and Z_A' as shown in Fig. 1.

Now (7)–(10) can be written

$$\frac{E_L}{E_A} = \frac{\overline{OZ_L'}}{\overline{OZ_A'}} e^{-\alpha d}, \quad (7')$$

$$\text{angle of } \frac{E_L}{E_A} = -\left(\frac{d}{\lambda} - \frac{a}{\lambda}\right) 360° \quad (7'')$$

$$\frac{I_L}{I_A} = \frac{\overline{BZ_L'}}{\overline{BZ_A'}} e^{-\alpha d} \quad (8')$$

$$\text{angle of } \frac{I_L}{I_A} = -\left(\frac{d}{\lambda} + \frac{b}{\lambda}\right) 360°, \quad (8'')$$

$$Z_A = Z_0 Z_A', \quad (9')$$

and

$$\eta = \frac{(\overline{OZ_L'})(\overline{BZ_L'}) \cos \theta_L}{(\overline{OZ_A'})(\overline{BZ_A'}) \cos \theta_A} e^{-2\alpha d} = \frac{(\overline{BZ_L'})^2 R_L}{(\overline{BZ_A'})^2 R_A} e^{-2\alpha d}. \quad (10')$$

The distances $\overline{OZ_L'}$, $\overline{OZ_A'}$, $\overline{BZ_L'}$, and $\overline{BZ_A'}$ are measured with any convenient scale. In (7''), the sign of a/λ is positive if the point Z_L' is above the line OZ_A'; otherwise it is negative. In (8''), the sign of b/λ is positive if the point Z_L' is above the line BZ_A'; otherwise it is negative.

If Z_0 and γd are not known, their values can be determined experimentally. Let Z_{sc} and Z_{oc} denote the values of Z_A when $Z_L = 0$ and ∞, respectively. Since

$$Z_{sc} = Z_0 \frac{1 - e^{-2\gamma d}}{1 + e^{-2\gamma d}} \quad (11)$$

and

$$Z_{oc} = Z_0 \frac{1 + e^{-2\gamma d}}{1 - e^{-2\gamma d}}, \quad (12)$$

it follows that

$$Z_0 = \sqrt{Z_{sc} Z_{oc}} \quad (13)$$

and

$$\sqrt{\frac{Z_{sc}}{Z_{oc}}} = \frac{1 - e^{-2\gamma d}}{1 + e^{-2\gamma d}}. \quad (14)$$

For convenience in notation, let $D = \sqrt{Z_{sc}/Z_{oc}}$. Now

$$e^{-2\gamma d} = \frac{1 - D}{1 + D}. \quad (15)$$

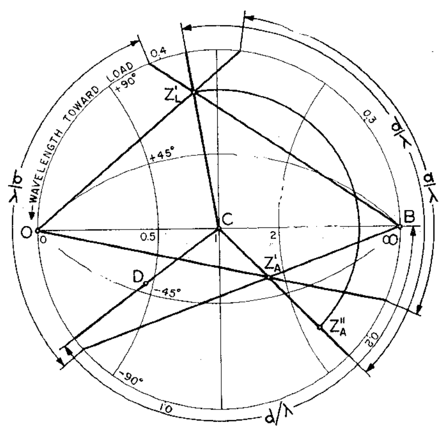

Fig. 1—Graphical construction.

When the point D is plotted on a transmission line chart as shown in Fig. 1, d/λ is measured as indicated and

$$e^{-2\alpha d} = \frac{\overline{CD}}{\overline{OC}}. \quad (16)$$

The values of the parameters for the construction shown in Fig. 1 are $Z_0 = 100/\underline{-10°}$, $Z_L = 85/\underline{65°}$, $d = 0.2\lambda$, and $e^{-\alpha d} = 0.707$. The results are $E_L/E_A = 0.63/\underline{-19°}$, $I_L/I_A = 1.24/\underline{-126°}$, $Z_A = 168/\underline{-42°}$, and $\eta = 0.45$.

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A Novel Technique for Making Precision Waveguide Twists

In a recent issue of the IRE Transactions on Microwave Theory and Techniques, Wheeler and Schwiebert[1] described step-twist waveguide components having good performance over a full waveguide band. These step-twist components are composed of a few short sections of straight rectangular waveguide, twisted about their common axis at their junction faces. Typically the length of each short section of the step-twist is the order of $\frac{1}{8}$ to $\frac{1}{4}$ of a guide wavelength, while the twist angle between sections may be as much as 30 degrees.

The technique[2] described here for constructing precision waveguide twists makes use of a large number of very short adjoining sections of waveguide. Hence it can be considered as the limiting case of Wheeler's and Schwiebert's step-twist technique. The twists constructed of these short sections of waveguide can easily be made to assume very complicated shapes that would be practically impossible to construct in any other manner. Furthermore, their electrical properties, such as vswr, attenuation, and power-handling capacity, are essentially the same as those of straight sections of waveguide.

The short lengths of waveguide used in these twists are stampings formed by a precision punch and die, and consequently can be inexpensively mass produced. Fig. 1 shows two typical stampings of 0.005-inch brass sheet suitable for X-band components.

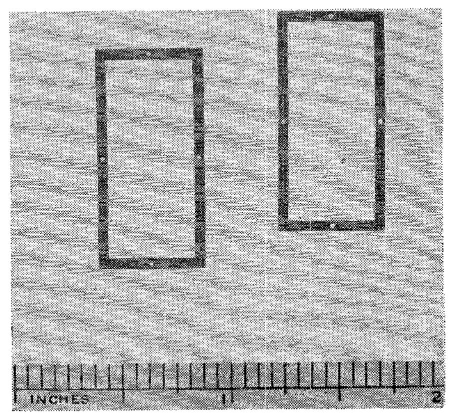

Fig. 1—Stampings used for constructing X-band precision twists.

One convenient method for aligning the stampings is to string them together on wires by means of the small holes shown in the figure. When the stampings are strung on the wires, the complete stack can then be twisted and one end translated with respect to the other in any desired manner, keeping the

[1] H. A. Wheeler and H. Schwiebert, "Step-twist waveguide components," Trans. IRE, vol. MTT-3, pp. 45–51; October, 1955.

[2] This construction technique was evolved by R. R. McPherson of Stanford Res. Inst. when called upon by the authors to make the multiple waveguide twist section described later in this letter.

two ends parallel to each other. The stack can then be compressed to assure good electrical contact between stampings. Good electrical contact can also be insured by soldering the stack, forming a solid unit which does not require continuous compression.

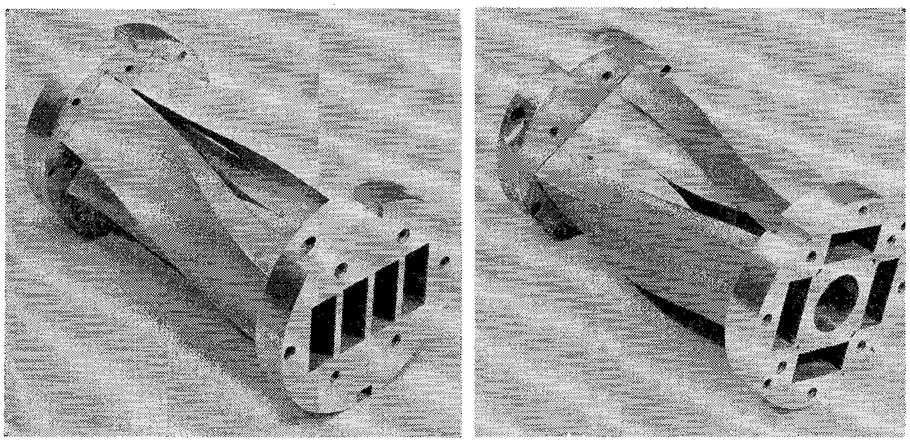

Fig. 2—Two views of a multiple waveguide twist section.

Fig. 2 shows two views of a very complex multiple waveguide twist section that was easily constructed of 0.005-inch thick brass stampings in the manner described above. In this case the assembly was soldered together in an electric oven. For the application in which this twist section was used, it was necessary that all the waveguides have the same electrical lengths to a high degree of precision. Initial measurements showed that over the 1.5 to 1 waveguide frequency range the electrical lengths of the four guides differed by as much as eight degrees due to their slightly different physical lengths. Therefore the widths of the appropriate individual guides were increased, by honing with an abrasive piston until all guides had electrical lengths within one degree of each other over this frequency range. The vswr of each guide was less than 1.02 over the full waveguide frequency range.

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On Symmetrical Matching

It is sometimes desirable to use symmetrical matching for a symmetrical lossless discontinuity in a transmission line or waveguide. A convenient form of such matching is two equal shunt susceptances placed across the transmission line, or waveguide, at positions which are symmetrical with respect to the discontinuity. The following procedure can be used to determine the positions and value of the shunt susceptances. 1) The position P and the value B of the shunt susceptance for one-sided matching is determined. (The susceptance may be inductive or capacitive; therefore, there are two possible pairs of P and B. Either of these pairs may be used.) 2) A shunt susceptance whose value is $B/2$ is placed at the position P. 3) The second shunt susceptance is placed on the other side of the discontinuity so that symmetry is restored.

It can be shown that no other positions or values of shunt susceptance can be used for this type of matching. When symmetry is not required, two unequal shunt susceptances may be used at the positions indicated by the above procedure if their sum is B.

The above procedure is not strictly valid unless it is assumed that the discontinuity, transmission line or waveguide, and shunt susceptances are lossless. However, this procedure can often be used to obtain satisfactory matching when the losses are small.

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